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We present modulation instability analysis including azimuthal perturbations of steady state CW propagation in multi-core fibre configurations with a central core. In systems with a central core, a steady CW evolution regime requires a power controlled phase matching offering interesting spatial division applications. Our results have general applicability and are relevant to a range of physical and engineering systems including high-power fibre lasers, optical transmission in multi-core fibre and systems of coupled nonlinear waveguides.

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The fast developing technology of optical multi-core fibre (MCF) provides for the possibility of spatial division multiplexing (SDM), enabling a scale-up in transmission capacity per-fibre that is a crucial challenge in optical communications [1, 2]. It is often assumed that though addition of spatial channels is technically very different from appending spectral channels, in terms of general consideration and system management, SDM, in many senses, is quite similar to technology of the wave-division multiplexing (WDM). One of the goals of this work is to point out that in MCF spatial channels may be very non-equal and transfer of channel management approaches from WDM to SDM is not always straightforward. For instance, in WDM signal-to-noise ratio (SNR) is often defined per spectral channel, assuming close to uniform spectral power density distribution between channels. As we will show below, in MCF configurations with a central core (e.g. 7-core MCF [1, 2]), a steady state CW propagation with equal power in all spatial channels is not possible at all, making in this system the regime with equal spatial channel powers inherently more prone to cross talks. Therefore, signal-to-noise ratio is SDM systems should be defined in a way different to standard WDM systems, taking into account difference between spatial channels.

Another important emerging application of multi-core fibre is in the field of high-power fibre lasers [3]. Nonlinear effects limit the power that can be transmitted in a single mode fibre. In a multi-core fibre, light in each core may be transmitted below threshold of the detrimental nonlinear effects while the total coherently combined power can be high. MCF technology is used in high brightness sources based on the coherent combining technique.

In both those major applications of MCF, nonlinear interactions between light in different cores can critically affect system performance. Therefore, knowledge of the limits imposed by the nonlinearity on coherent transmission of light through the MCF is of high practical importance. Despite this, the nonlinear dynamics of light in MCF is not yet well studied.

The mathematical analysis of nonlinear wave propagation in multi-core fibres is also a generic problem with numerous links to the theory of nonlinear discrete systems (see e.g. [4–7, 9–12] and references therein). As was already demonstrated in [4] in MCF with non-equal cores (the most simple and general case is N peripheral cores surrounding the central core), phase matching and stable coherent propagation are possible only due to nonlinear effects for a certain power balance between cores. In [4] the stability problem of steady-state propagation was solved in the radial approximation without consideration of azimuthal perturbations. It has been shown in [4] that, surprisingly, even at high light intensities, stable coherent propagation is possible. In this Letter we extend the stability analysis to the important case of azimuthal perturbations, and we account for the possibility of power transfer between peripheral cores. In the optical communication context, our results provide the underlying theory explaining why in the spatial-division-multiplexing technique with MCF having a central core, spatial power distribution (power per spatial channel) should not be uniform, but instead has to be adjusted to the multi-core fibre geometry, as described below.

The basic model considered here is a version of the discrete nonlinear Schrödinger equation:

$$i\frac{\partial A_k}{\partial z} + \sum_{m=0}^{N} C_{km}A_m + 2\gamma_k|A_k|^2 A_k = 0;\ k = 0, ..., N \quad (1)$$

Here A_k is the field in the k-th core ($k = 1, 2, 3, .., N$), with A_0 being the field in the central core; C_{mk} is the coupling coefficient between modes m and k; $C_{k,k\pm1} = C_1 (k \neq 0), C_{k,0} = C_0$, $C_{kk} = \beta_k$. The coefficients related

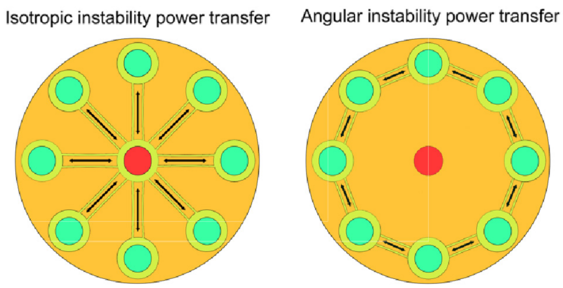

Fig. 1. The schematic depiction of the multi-core fibre and power exchanges between cores.

to wave numbers and nonlinearity in peripheral cores, without loss of generality, are assumed to be equal ($\beta_k = \beta_1$ and $\gamma_k = \gamma_1$). We assume though that, in general, nonlinear coefficients in central and peripheral cores can be different. It is convenient to re-write these equations in normalized units:

$$i\frac{\partial U_0}{\partial z} + \frac{1}{N}\sum_{k=1}^{N} U_k + \frac{2N\gamma_0}{\gamma_1}|U_0|^2 U_0 = 0, \tag{2}$$

$$i\frac{\partial U_k}{\partial z} + (\kappa - 2C)U_k + C(U_{k+1} + U_{k-1}) + U_0 + 2|U_k|^2 U_k = 0, \tag{3}$$

Here we introduced normalized variables: $A_{0,k} = \sqrt{P_{0,1}}\, U_{0,k}\, e^{i\beta_0 L z}$; $z' = z/L$; $L = 1/(C_0\sqrt{N})$, $C = C_1/(C_0\sqrt{N})$, $P_0 = NP_1 = N^{3/2}C_0/\gamma_1, \kappa = (\beta_1 - \beta_0 + 2C_1)/(C_0\sqrt{N})$. The total (normalized by P_0) power $P_t = N(|U_0|^2 + |U_1|^2)$, is a conserved quantity.

As was pointed out in [4] in the case of multiple peripheral cores surrounding a central core, even the existence of steady state solution is nontrivial. To provide for coherent steady state CW light evolution in multiple cores, the difference in propagation constants has to be compensated by the nonlinear phase shifts:

$$\{U_0, U_k\} = \{A, B\} \times e^{i\lambda z}, \Gamma = \frac{B}{A}, \tag{4}$$

$$|A|^2 = \frac{P_t}{N(1+\Gamma^2)},\ \lambda = \Gamma + \frac{2\gamma_0 P_t}{\gamma_1(1+\Gamma^2)}. \tag{5}$$

$$\Gamma^4 - \left(\kappa + \frac{2P_t}{N}\right)\Gamma^3 - \left(\kappa - \frac{2\gamma_0 P_t}{\gamma_1}\right)\Gamma - 1 = 0. \tag{6}$$

In this Letter we limit analysis only by the case of in- and out-of-phase fields A and B, meaning real values of Γ. More general case will be presented elsewhere. The steady state solutions in such system is possible only with a certain imbalance (given by the factor $\Gamma^2 = B^2/A^2$) between powers propagating in central (A) and ring (B) cores. The physics of this effect is rather transparent - the power split is due to the nonlinear phase shift contribution to the phase matching condition required for coherent propagation in multiple cores.

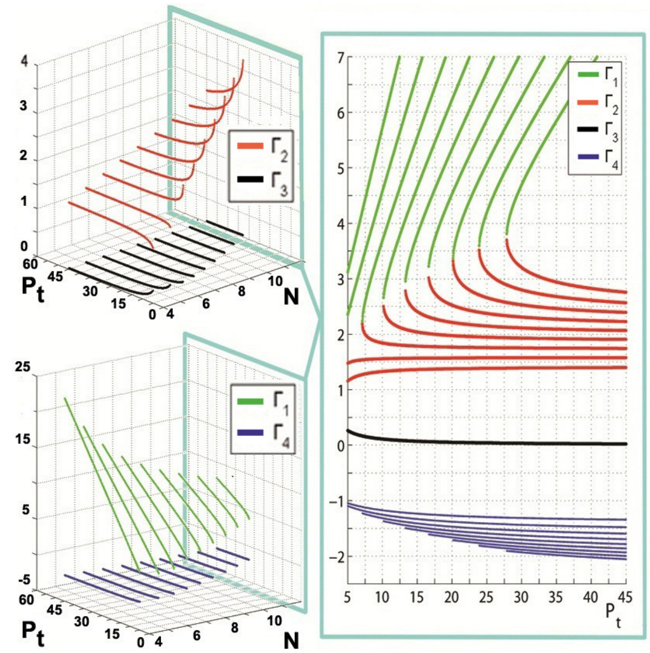

Fig. 2. Four values of Γ corresponding to different power splits between cores as functions of total input power; here $\gamma_0/\gamma_1 = 0.5$ and $\kappa = 1$. Here different curves for each branch correspond to N varying from 3 to 12 (from the bottom to the top, from left to right for green curve).

The amount of power that has to be coupled to each core for steady state evolution given by solutions of (6) depends on four parameters: (i) N, (ii) input power P_{in} (or total power P_t), (iii) the linear phase mismatch κ, and (iv) the ratio between the nonlinear coefficients γ_0/γ_1. To get the idea of the solution structure, consider the practically important case $P_t >> 1$. In this case, from (6) we will get explicitly four families of solutions. In the $\Gamma_1 = 2P_t/N$ and $\Gamma_3 = \gamma_1/(2\ \gamma_0\ P_t)$ most of the energy propagates in the ring or central core, respectively. For $\Gamma_{2,4} = \pm\sqrt{\gamma_1 N/\gamma_0}$ the ratio of energy in the ring and the central core is independent of the propagating power. Negative Γ means out-of phase fields in the central and peripheral cores.

Consider now the stability of steady state solutions of (4-6) - the analogue of the modulation instability for a low dimension discrete system. The small amplitude disturbance is taken in a standard form $\{U_0, U_k\} = \{A + a + ib, B + c + id + (f + ih)e^{isk}\} \times e^{i\lambda z}$ and $k = 1, 2, 3, ..., N$. It is easy to see that perturbations of U_k proportional to $\exp[pz]$ have an isotropic (k-independent) part (considered in [4]) and an angular (k-dependent) contribution. Straightforward analysis shows that the angular part of perturbations is independent of the isotropic part, and the increment (growth rate) of instability due to angular perturbations is:

$$p_l^2 = (\frac{1}{\Gamma_l} + 4C\sin^2[\frac{s}{2}])(\frac{4\Gamma_l^2 P_t}{N(1+\Gamma_l^2)} - \frac{1}{\Gamma_l} - 4C\sin^2[\frac{s}{2}]), \tag{7}$$

here $l = 1, 2, 3, 4$ corresponds to four branches of CW steady states. Recall that $C = C_1/(C_0\sqrt{N})$.

From periodicity condition $s = 2\pi m/N$ with $m = 1, 2, 3, ..., N$. Note that the expression for the increment Eq. 7 looks structurally exactly as the classical formula for MI growth rate [13, 14]: $p_l^2 = \Lambda(\alpha P_t - \Lambda)$. However, here $\Lambda = 1/\Gamma_l + 4C\sin^2[s/2]$ is a discrete variable. The Eq. 7 without the term $1/\Gamma_l$ describes the modulation instability (MI) in discrete systems (see e.g. [7, 8, 12]) and in the limit $s \ll 1$ transforms into conventional MI growth rate [13, 14]. We see that the effect of a central fiber $1/\Gamma$ plays a stabilizing role for positive Γ. For negative Γ, mode Γ_4 - the presence of the central core enhance the instability. We discuss below only the new features introduced by the possibility of a positive Γ.

The minimal value of $s = 2\pi/N$, $m = 1$ gives the threshold (in power) of the azimuthal modulation instability:

$$\frac{4\Gamma_l^2 P_t}{N(1+\Gamma_l^2)} < \frac{1}{\Gamma_l} + 4C\sin^2[\frac{\pi}{N}], \tag{8}$$

Here $\Gamma_l = \Gamma_l(N, P_t)$ is a function of N and P_t making this equation an implicit condition on power. For continuous case $N \to \infty$ MI has no threshold. However, both the discreteness and presence of a central core (with positive Γ) suppress the instability. The maximum growth rate p_{max} is reached at:

$$\frac{2\Gamma_l^2 P_t}{N(1+\Gamma_l^2)} = \frac{1}{\Gamma_l} + 4C\sin^2[\pi m/N], \tag{9}$$

and is $p_{max} = 2\Gamma_l^2 P_t/[N(1+\Gamma_l^2]$, The instability is developed at the length $L \sim 1/p_{max}$.

Analysis of the analytical asymptotic solutions (valid at $P_t >> 1$) shows that only Γ_3 is a stable solution (in some range of parameters in the plane (N, P_t) shown by color in Fig. 3). All other solutions $\Gamma_{1,2,3}$ are unstable. In the case of angular perturbations it is not possible to derive an exact analytical expressions defining the stability zones for all values of P_t. The stability boundaries for P_t not large can be found numerically. The calculations the sufficient conditions of stability and instability for $\Gamma_3(N, P_t)$ and, thus, define stable and unstable areas in the plane (N, P_t) as illustrated by Figure 3: The threshold curve (8) for Γ_3, when big fraction of energy is concentrated in the central core is presented in Fig. 3. The increment of instability p^2 as a function of s for Γ_2 and Γ_3 is depicted in Fig. 4 for various N and P_t. We see that for the fixed P_t the growth rate is decreases with the number of cores. The intensity in every core is going down with N increase and modulation instability is determined by the local radiation intensity. However, one can see that the growth decrease is slower then 1/N which indicates that the increase in core numbers makes MCF more susceptive to the modulation instability.

The maximum power P_{max} that can be transported in a single core in unrelated to the threshold power P_{th}

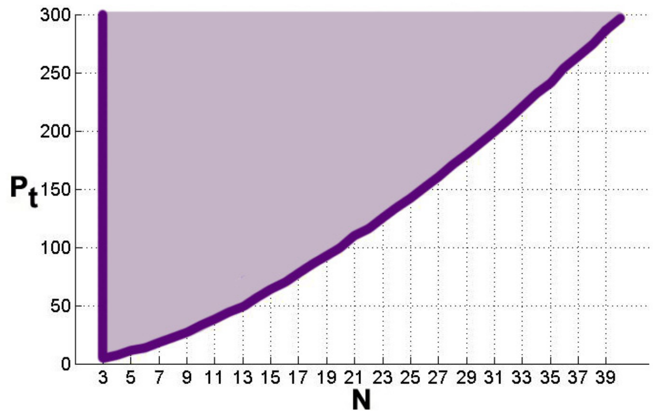

Fig. 3. Angular stability area (shown by colour) of Γ_3 (Γ_1, Γ_2 and Γ_4 are unstable) corresponding to different values of the total power P_t and N; here $\gamma_0/\gamma_1 = 1$ and $\kappa = 1$

for the instability calculated above. Hence, our analysis paves the way for design of MCF that will support the stable propagation of the total power P_t well above P_{max}. The coherent output from the fiber end can be then combined in one beam leading to the compact and efficient beam combining scheme.

Nonlinear instability leads to periodic exchange of power between cores. With a growing number of cores these oscillations may become stochastic. As a result, the instability makes power dynamics uncontrollable. Therefore, knowledge of the criteria of instability onsets is important for design of physical systems and devices based on MCF. The considered instability is an extreme discrete limit of the conventional modulation instability in the continuous media and discrete systems (see e.g. [7,12–14] and references therein). Due to the generality of the master equations, we anticipate that our results may provide new outlook at the traditional arrays of coupled nonlinear waveguides [5–7, 9–12] We would like to stress also that though the presented analysis deals with the CW propagation, the obtained results are applied to time dependent fibre communication channels. In that case, the power should be understood as time-averaged signal characteristics, such as the average signal power. Note also that the presented theory can be generalized in straightforward manner to pulse propagation in MCF. As a matter of fact, in [15] the efficiency of nonlinear matching through a fundamental soliton coupling from one fibre into another was studied that has similarity with CW power matching studied here. This paves a way to numerous applications, for instance, a controlled Raman red-shift and supercontinuum generation. To conclude, in this Letter we have presented a theory of an instability including azimuthal perturbations of a steady state CW propagation in multi-core fibre configurations with a central core. In MCF with a central core, a steady state CW propagation requires a power supported phase matching. This has an important consequence for spa-

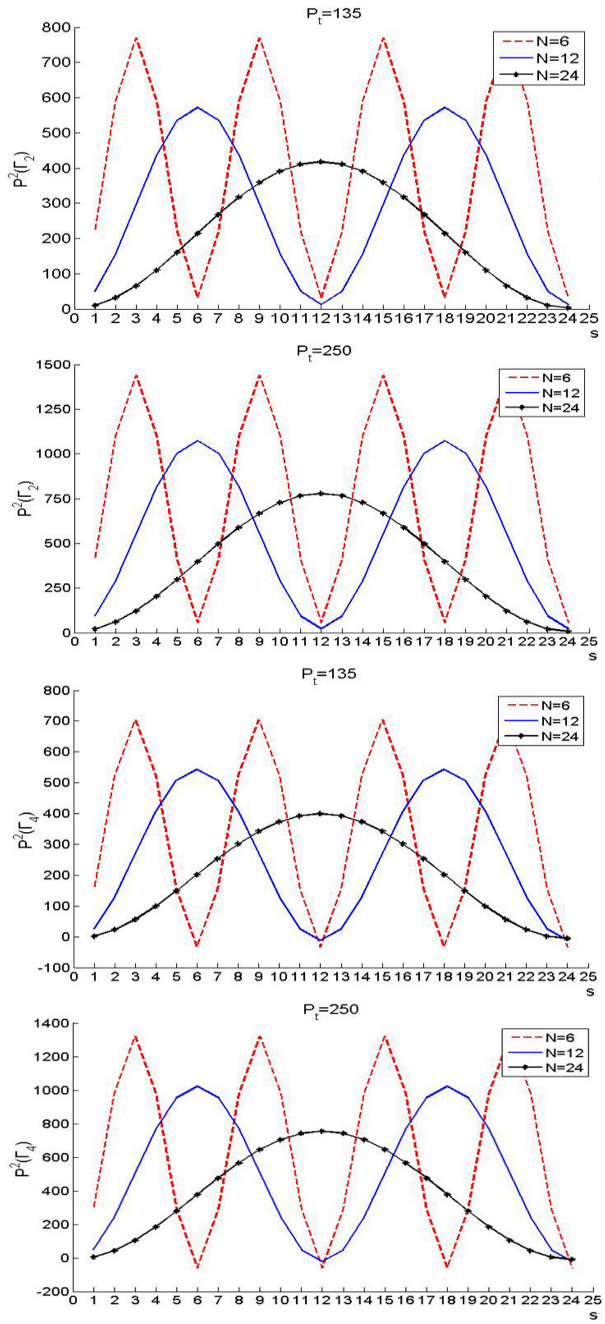

Fig. 4. Increment of instability p^2 as a function of s for Γ_2 and Γ_4 corresponding to different P_t and N (red dashed line - $N = 6$; solid blue - $N = 12$ and solid-dotted black - $N = 24$).

tial division multiplexing. In the context of optical fibre communications using MCF our results can be understood in the following way. In the considered system of multi-core fibres with a central core, a power per spatial channel cannot be uniform in a stable propagation. Stable propagation requires certain disbalance between the power in a central core and other given by Γ_l^2. This disbalance depends on MCF geometry and other system parameters as described above. In particular, this means that signal-to-noise ratio should not be introduced per spatial channel and more sophisticated definitions are required. The developed theory is rather generic and has a number of applications from high power fibre lasers to bulk nonlinear wave-guiding systems.

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